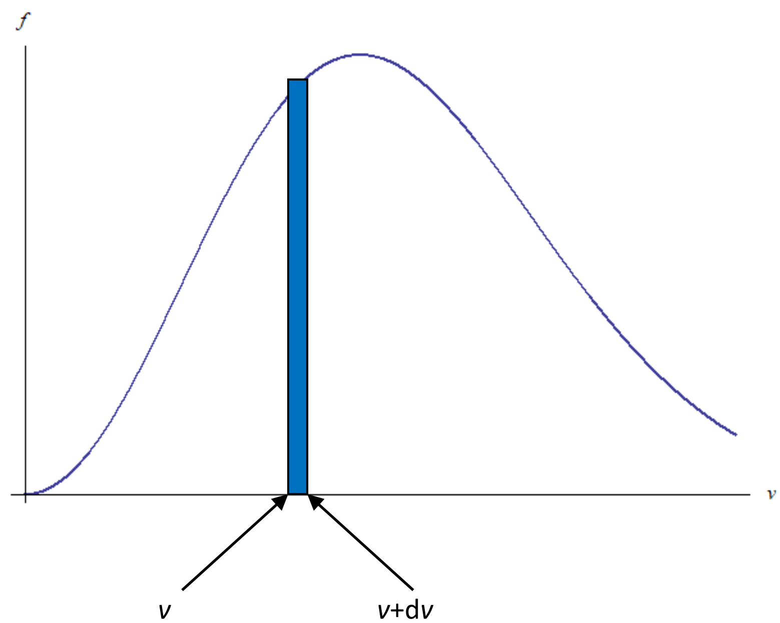
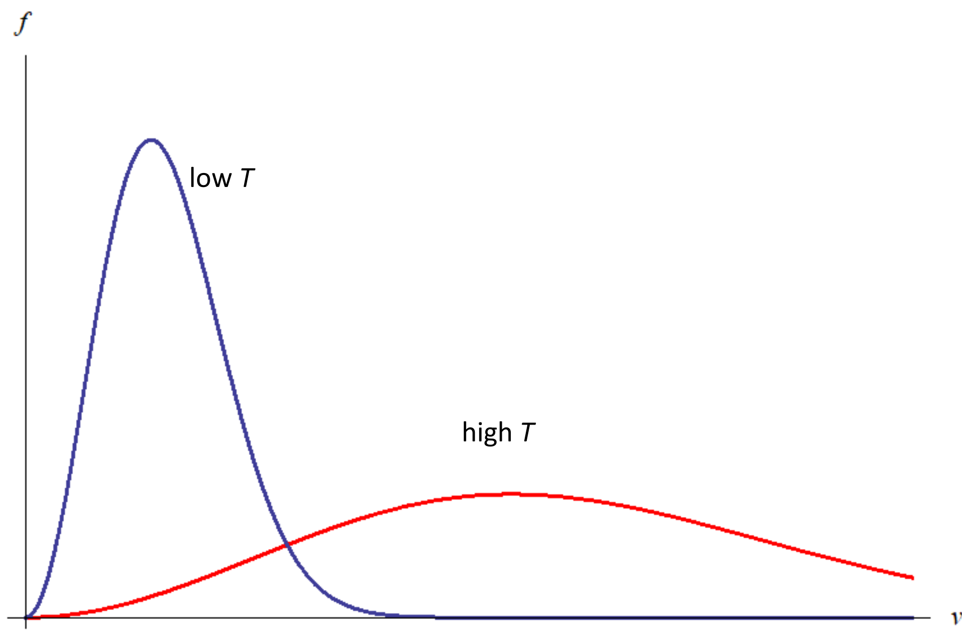
Teacher notes
Topic B

Another view of entropy

We get another view of entropy by considering the Maxwell-Boltzmann distribution. The quantity $f\,\mathrm{d}v$ gives the number of molecules which have a speed in the interval v to $v+\mathrm{d}v$. This corresponds to the area of the thin rectangle in the graph below.



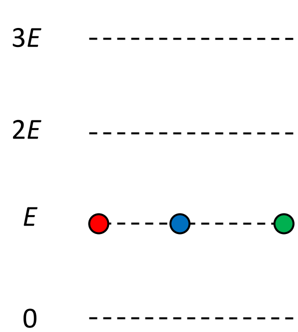
Shown in the next graph are two distributions for the same ideal gas at different temperatures. The red curve corresponds to the high temperature. We observe that at high temperature **the spread in speed (and hence kinetic energy) is much larger**. The entropy of the gas at the high temperature is greater than that at low temperature: the gas was heated to reach the high temperature and so heat was provided to it increasing the temperature and entropy.



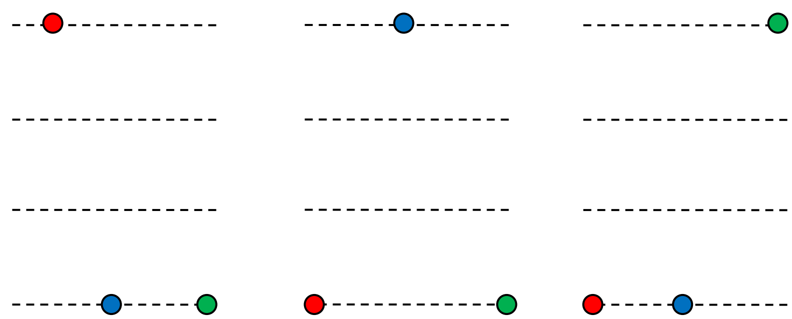
In the textbook we discussed the statistical interpretation of entropy in terms of the number of microstates available to a system: the larger the number of microstates the larger the entropy. Here we see that greater entropy corresponds to a greater spread in energy. How do we connect the two views of entropy?

Imagine that we have three molecules, and we have a total energy 3*E* that we will distribute among the three. Assume the energy is quantized in units of *E* so that we can only offer integral multiples of *E* each time (0, 1, 2 or 3).

One way is to do give an amount *E* to each molecule. There is only one way this can be done. The molecules have the same energy and so zero spread. The entropy is low because the energy spread is low (zero). We get the same conclusion in terms of microstates. There is only one microstate in which the molecules have the same energy. This is shown below.

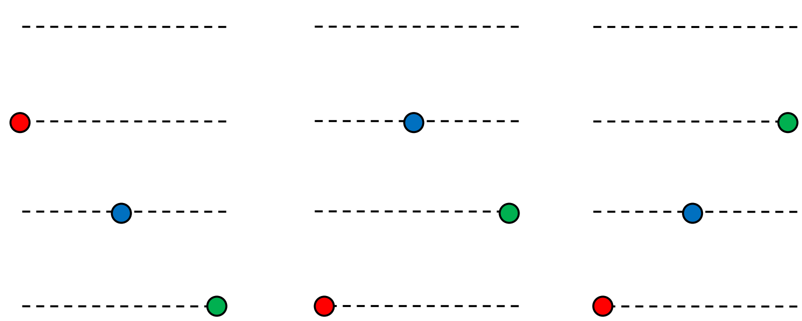


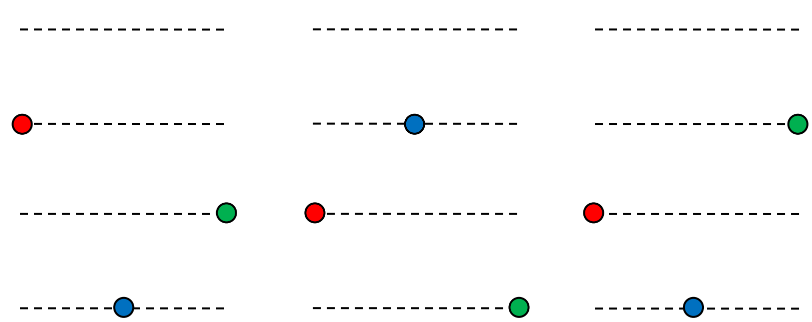
A configuration with non-zero spread is shown below:



One molecule gets all the energy (3*E*) and the other two get zero. There are three microstates corresponding to this configuration. The entropy is higher than that of the previous configuration. The spread in energy is higher as well. These three microstates belong to the same configuration because they are equivalent, they would be identical if we had not colored the molecules.

Finally, consider the following configuration consisting of six microstates. They also belong to the same configuration because they are identical apart from the coloring of the molecules.





The spread in energy is large and the number of microstates is large. Either way, the entropy is large.

So, we see that entropy is associated with large numbers of microstates and equivalently with a greater spread in how the energy is distributed among the molecules of the system. And finally, it is also consistent with the vague notion of entropy as a measure of “disorder”. The state with molecules having different energies is more disordered than the state in which all molecules have the same energy. This is clear in the example with just 3 molecules and becomes more apparent as the number of molecules increases to a very large number.